UDC 539.3

INTERACTION OF A DOUBLY-PERIODIC SYSTEM OF RECTILINEAR CRACKS IN AN ISOTROPIC MEDIUM

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The problem of the tension and shear on a plane isotropic medium weakened by a doubly-periodic system of rectilinear cracks is considered. General representations are constructed for the solutions describing a class of problems with doubly-periodic stress distribution outside the cracks. The fundamental singular equation of the problem is reduced to an infinite system of linear algebraic equations without the intermediate step of reducing it to a Fredholm equation. The procedure to determine the stress intensity factors is written out.

Questions related to modeling the described lattice by a continuous isotropic medium are examined and the elastic characteristics of this latter are determined (the macroscopic parameters of a medium with cracks). Results of computations are presented.

The doubly-periodic problem for a symmetric rhombic lattice has been considered by another method in [1, 2].

1. Formulation of the problem. Let the mean stresses S_1, S_2 and S_{12} (Fig. 1) act in an unbounded isotropic plate weakened by a doubly-periodic system of

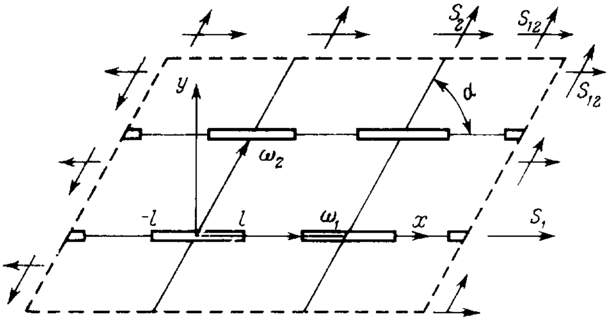

Fig. 1

rectilinear cracks. We assume that identical self-equilibrated loads

$$F^{\pm}(x) = \sigma_y^{\pm}(x) + i\tau_{xy}^{\pm}(x), \qquad x \in [-l, l]$$

are given at congruent, Hölder-continuous, points on the crack edges. Let

$$\omega_1, \omega_2 \ (\operatorname{Im} \omega_1 = 0, \ \operatorname{Im} \omega_2 / \omega_1 > 0)$$

denote the fundamental periods, and D the domain occupied by the plate material. We place the beginning and ending of the cracks, respectively, at the points

$$-l + m\omega_1 + n\omega_2, \quad l + m\omega_1 + n\omega_2$$
$$(m, n = 0, \pm 1, \pm \ldots; 0 \leqslant l < \omega_1 / 2)$$

By virtue of the symmetry of the boundary conditions and the geometry of the domain D, the stresses in D are doubly-periodic functions with the fundamental periods ω_1 and ω_2.

Following [3], we introduce a function analytic in D

$$\overline{\Omega}(z) = \Psi(z) + \Phi(z) + z\Phi'(z) \tag{1.1}$$

In combination with the analytic function $\Phi(z)$, this function uniquely determines the stresses and displacements in the domain under consideration. We have [3]

$$\begin{aligned}
&\sigma_x + \sigma_y = 4\mathrm{Re}\,\Phi(z) \\
&\sigma_y - \sigma_x + 2i\tau_{xy} = 2\{\overline{\Omega}(z) - \Phi(z) - (z - \bar{z})\Phi'(z)\} \\
&\sigma_y + i\tau_{xy} = \overline{\Omega}(z) + \overline{\Phi(z)} - (z - \bar{z})\Phi'(z) \\
&2G(u - iv) = \varkappa\overline{\varphi(z)} - \overline{\omega}(z) - (\bar{z} - z)\Phi(z), \qquad \overline{\omega}(z) = \int \overline{\Omega}(z)dz
\end{aligned} \tag{1.2}$$

Conditions for periodicity of the stresses

$$\Phi(z + \omega_\nu) = \Phi(z), \ \overline{\Omega}(z + \omega_\nu) - \overline{\Omega}(z) = (\omega_\nu - \overline{\omega}_\nu)\Phi'(z) \qquad \nu = 1, 2 \tag{1.3}$$

follow from (1.2).

It is expedient to construct the functions $\overline{\Omega}(z)$ and $\Phi(z)$ in such a manner that, firstly, a given jump in the combination $\sigma_y + i\tau_{xy}$ would be assured in going from one crack edge to its opposite, and secondly, the periodicity conditions (1.3) would be satisfied automatically. The desired representations are

$$\Phi(z) = \frac{1}{2\pi i}\int_{-l}^{l} p(x)\zeta(x - z)\,dx + A \tag{1.4}$$

$$\overline{\Omega}(z) = \frac{1}{2\pi i}\int_{-l}^{l} \{F(x) - \overline{p(x)}\}\zeta(x - z)\,dx +$$
$$\frac{1}{2\pi i}\int_{-l}^{l} p(x)\{\rho_1(x - z) - (x - z)\rho(x - z) + \zeta(x - z)\}\,dx + B$$

$$F(x) = F^+(x) - F^-(x)$$

Here $\rho(z)$ and $\zeta(z)$ are Weierstrass functions [4], $\rho_1(z)$ is a special meromorphic function [5, 6], $F(x)$ is the jump in the expression $\sigma_y(x) + i\tau_{xy}(x)$ in $[-l, l]$, A and B are constants governed by the static conditions, and $p(x)$ is the desired, generally complex, function in $(-l, l)$. Let us append an additional equality expressing the condition that the displacements are unique in D

$$\int_{-l}^{l} p(x)\,dx = 0 \tag{1.5}$$

to the representations (1.4). Compliance of the representations (1.4) with the conditions

(1.3) follows from the quasi-periodicity of $\zeta(z)$, the equality (1.5) and the relationships [5]

$$\rho_1(z+\omega_\nu)-\rho_1(z)=\bar{\omega}_\nu\rho(z)+\gamma_\nu, \qquad \nu=1,2 \tag{1.6}$$

$$\gamma_\nu=2\rho_1\left(\frac{\omega_\nu}{2}\right)-\bar{\omega}_\nu\rho\left(\frac{\omega_\nu}{2}\right)$$

The specified jump in the function $\sigma_y+i\tau_{xy}$ in $[-l, l]$ is also assured. This is easily detected if the continuity of the kernel $\rho_1(x-z)-(x-z)\rho(x-z)+\zeta(x-z)$ in the fundamental period parallelogram is táken into account. Under the condition (1.5) the representations (1.4) must assure the existence of the given mean stresses S_1, S_2 and S_{12} in D. To this end, let us determine the constants A and B from the relationships

$$g(z+\omega_2)-g(z)=i\mid\omega_2\mid(S_1+S_{12}e^{i\alpha}), \quad \alpha=\arg\omega_2 \tag{1.7}$$

$$g(z+\omega_1)-g(z)=-i\omega_1(S_{12}+S_2e^{i\alpha})$$

$$g(z)=\varphi(z)+z\overline{\Phi(z)}+\overline{\psi(z)}=\varphi(z)+\omega(\bar{z})+(z-\bar{z})\overline{\Phi(z)}$$

$$\varphi(z)=\int\Phi(z)\,dz, \qquad \psi(z)=\int\Psi(z)\,dz$$

The combination $g(z)$ turns out to be a quasi-periodic function. Its increment at the fundamental periods is found by taking account of the following properties of the functions in $g(z)$ [4, 6]:

$$\sigma(z+\omega_\nu)=-\sigma(z)\exp\left\{\delta_\nu\left(z+\frac{\omega_\nu}{2}\right)\right\}, \qquad \nu=1,2 \tag{1.8}$$

$$\delta_\nu=\zeta\left(z+\frac{\omega_\nu}{2}\right)-\zeta(z)=2\zeta\left(\frac{\omega_\nu}{2}\right)$$

$$\zeta_1(z+\omega_\nu)-\zeta_1(z)=\bar{\omega}_\nu\zeta(z)-\gamma_\nu z-\gamma_\nu^*$$

$$\zeta_1(z)=-\int_0^z\rho_1(z)\,dz$$

It is assumed here that the point z lies in the fundamental period parallelogram containing the origin, and γ_ν^* is an insignificant constant in our case. By virtue of (1.4), (1.5) and (1.8) we arrive from (1.7) at the system of equations

$$(A+\bar{A})\bar{\omega}_1+(B-A)\omega_1+a(\delta_1+\gamma_1)+\bar{a}(\delta_1+\bar{\delta}_1)= \tag{1.9}$$
$$i\omega_1(S_{12}+S_2e^{-i\alpha})-\delta_1 f$$

$$(A+\bar{A})\bar{\omega}_2+(B-A)\omega_2+a(\delta_2+\gamma_2)+\bar{a}(\delta_2+\delta_2)=$$
$$-i\mid\omega_2\mid(S_1+S_{12}e^{-i\alpha})-\delta_2 f$$

$$a=\frac{1}{2\pi i}\int_{-l}^{l}xp(x)\,dx, \qquad f=\frac{1}{2\pi i}\int_{-l}^{l}xF(x)\,dx$$

Taking account of the equalities [4, 5]

$$\delta_1\omega_2-\delta_2\omega_1=2\pi i, \qquad \gamma_1\omega_2-\gamma_2\omega_1=\delta_2\bar{\omega}_1-\delta_1\bar{\omega}_2$$

let us represent the solution of the system (1.9) as

$$\operatorname{Re}A=\frac{1}{4}(\sigma_1+\sigma_2)-\frac{\pi}{2S}f-\frac{1}{\omega_1}\operatorname{Re}(a\delta_1), \qquad S=\omega_1 H \tag{1.10}$$

$$B - A = \frac{1}{2}(\sigma_2 - \sigma_1) + i\tau + \frac{\pi - H\delta_1}{S} f - \frac{a\gamma_1}{\omega_1} - \frac{\bar{a}\delta_1}{\omega_1}$$
$$\sigma_2 = S_2 \sin \alpha, \quad \tau = S_{12} + S_2 \cos \alpha, \quad H = \operatorname{Im} \omega_2$$
$$\sigma_1 \sin \alpha = S_1 + 2S_{12} \cos \alpha + S_2 \cos^2 \alpha$$

Here σ_1, σ_2 and τ are the mean stresses in areas perpendicular to the ox and oy coordinate axes. The compatibility condition of the system (1.9) is

$$\operatorname{Im} f = \operatorname{Im} \left\{ \frac{1}{2\pi i} \int_{-l}^{l} xF(x)\,dx \right\} = \frac{1}{2\pi} \int_{-l}^{l} (\sigma_y^+ - \sigma_y^-)\,x\,dx = 0 \qquad (1.11)$$

Condition (1.11) is satisfied automatically because of the self-equilibration of the load given on the edges of the cracks.

Therefore, under the conditions (1.5) the representations (1.4) describe a class of doubly-periodic problems for a plane with cracks.

2. Algorithm of the solution of the problem. The stress periodicity conditions are satisfied because of the selection of the representations of the desired functions, hence it is sufficient to satisfy the boundary conditions just on the edges of the fundamental crack. These boundary conditions are

$$\overline{\Omega}(x) + \overline{\Phi(x)} = \begin{cases} F^+(x_0), & x = x_0 + 0i, \quad x_0 \in [-l, l] \\ F^-(x_0), & x = x_0 - 0i \end{cases} \qquad (2.1)$$

Because of the very construction of the functions (1.4), it is sufficient to satisfy one of the conditions (2.1) since the realization of the second of the conditions (2.1) results in the same singular equation. Passing to the limit in (1.4) and substituting the limit values of the functions $\overline{\Phi(z)}$ and $\overline{\Omega}(z)$ obtained into one of the boundary conditions (2.1), we arrive at a singular equation in $p(x)$

$$\frac{1}{2\pi i} \int_{-l}^{l} \overline{p(x)} \{\zeta(x - x_0) + \overline{\zeta(x - x_0)}\}\,dx - \qquad (2.2)$$
$$\frac{1}{2\pi i} \int_{-l}^{l} p(x) \{\rho_1(x - x_0) - (x - x_0)\rho(x - x_0) + \zeta(x - x_0)\}\,dx -$$
$$B - \bar{A} = \frac{1}{2\pi i} \int_{-l}^{l} F(x)\zeta(x - x_0)\,dx - \frac{F^+(x) + F^-(x)}{2}$$

This equation (more accurately, system of equations) is easily reduced to customary form if expansions in the main period parallelogram are kept in mind [7]

$$\zeta(z) = \frac{1}{z} - \sum_{j=1}^{\infty} \frac{g_{j+1}^* z^{2j+1}}{\omega_1^{2j+2}}, \qquad g_k^* = {\sum_{m,n}}' \frac{1}{T^{2k}} \qquad (2.3)$$
$$\rho(z) = \frac{1}{z^2} + \sum_{j=1}^{\infty} \frac{(2j+1)\, g_{j+1}^*}{\omega_1^{2j+2}} z^{2j}, \qquad \rho_k^* = {\sum_{m,n}}' \frac{\bar{T}}{T^{2k+1}}$$
$$\rho_1(z) = \sum_{j=1}^{\infty} \frac{(2j+2)\,\rho_{j+1}^*}{\omega_1^{2j+2}} z^{2j+1}$$
$$T = m + n\frac{\omega_2}{\omega_1}, \qquad m, n = 0, \pm 1, \pm, \ldots; \; k = 2, 3, \ldots$$

After simple computations we obtain

$$\frac{1}{\pi i}\int_{-1}^{1}\frac{\theta(\xi)}{\xi-\xi_0}d\xi-\frac{1}{\pi i}\int_{-1}^{1}\theta(\xi)K(\xi-\xi_0)d\xi- \tag{2.4}$$
$$\frac{1}{\pi i}\int_{-1}^{1}\overline{\theta(\xi)}K_*(\xi-\xi_0)d\xi = {}^{\backprime}H(\xi_0)$$

$$\theta(\xi)=p(x),\quad \xi=\frac{x}{l},\quad \xi_0=\frac{x_0}{l},\quad \lambda=\frac{2l}{\omega_1},\quad -1<\xi_0<1$$

$$K(\xi)=\sum_{j=0}^{\infty}K_j\left(\frac{\lambda}{2}\right)^{2j+2}\xi^{2j+1},\qquad 0\leqslant\lambda<1$$

$$K_*(\xi)=\sum_{j=0}^{\infty}K_j^*\left(\frac{\lambda}{2}\right)^{2j+2}\xi^{2j+1},\qquad K_0=\omega_1\,\mathrm{Re}\,\delta_1$$

$$K_j=\mathrm{Re}\,g_{j+1}^{*},\quad K_0^*=-\frac{\omega_1}{2}(\bar{\gamma}_1+\bar{\delta}_1),\quad K_j^*=(j+1)\times$$
$$(\bar{\rho}_{j+1}-\bar{g}_{j+1}),\quad j=1,2,\ldots$$

$$H(\xi_0)=i\tau-\sigma_2+\overline{F_1(x_0)}+\frac{1}{2\pi i}\int_{-l}^{l}\overline{F(x)}\left\{\overline{\zeta(x-x_0)}-\frac{x\bar{\delta}_1}{\omega_1}\right\}dx$$

$$F(x)=F^+(x)-F^-(x),\quad F_1(x)=\frac{1}{2}\{F^+(x)+F^-(x)\}$$

The additional condition (1.5), represented in the form

$$\int_{-1}^{1}\theta(\xi)\,d\xi=0 \tag{2.5}$$

must be added to the singular equation (2.4).

If the lattice and the external load are symmetric relative to the coordinate axes, then (2.4) degenerates into one singular equation in the function $\theta(\xi)$ which takes on pure imaginary values. In the general case, (2.4) is a system of two singular equations in the complex function $\theta(\xi)$.

Let us reduce (2.4) to an infinite system of linear algebraic equations by skipping the intermediate step ofregularization and reduction to a Fredholm equation of the second kind. We assume

$$\theta(\xi)=\frac{\theta_0(\xi)}{\sqrt{1-\xi^2}} \tag{2.6}$$

Where $\theta_0(\xi)$ is a Hölder-continuous function in $[-1, 1]$. Let us seek $\theta_0(\xi)$ as a series of Chebyshev polynomials of the first kind. We have

$$\theta_0(\xi)=\sum_{k=1}^{\infty}A_kT_k(\xi),\qquad T_k(\xi)=\cos(k\arccos\xi) \tag{2.7}$$

It is seen directly that the additional condition (2.5) is hence satisfied automatically.

The relationships for the Chebyshev polynomials of the first and second kinds $T_k(\xi)$ and $U_k(\xi)$ are presented below

$$\int_{-1}^{1} \frac{T_k(\xi)}{\sqrt{1-\xi^2}} \frac{d\xi}{\xi-\xi_0} = \pi U_{k-1}(\xi_0), \quad k = 1, 2, \ldots \tag{2.8}$$

$$\int_{-1}^{1} (1-\xi)^s T_k(\xi) \frac{d\xi}{\sqrt{1-\xi^2}} = \frac{\pi (2s)!}{2^s (k+s)!\,\Gamma(s-k+1)} = \pi b_{k,s}, \quad s, k = 0, 1, \ldots$$

$$\int_{-1}^{1} (1+\xi)^s \sqrt{1-\xi^2} U_k(\xi)\, d\xi = \frac{\pi (2s+1)!\,(k+1)}{2^s (k+s+2)!\,\Gamma(s-k+1)} = \pi a_{k,s}$$

$$U_k(\xi) = \frac{\sin[(k+1)\arccos\xi]}{\sin(\arccos\xi)}$$

Substituting (2.6), (2.7) into the integral equation (2.4), using the first formula in (2.8) and the orthogonality of the functions $U_k(\xi)$ in $[-1, 1]$, we arrive at an infinite system of linear algebraic equations in the coefficients A_k

$$A_{k+1} - \sum_{n=1}^{\infty} C_{nk} A_n - \sum_{n=1}^{\infty} C_{nk}^* \bar{A}_n = iH_k, \quad k = 0, 1, \ldots \tag{2.9}$$

$$C_{nk} = \frac{2}{\pi^2} \int_{-1}^{1}\int_{-1}^{1} \frac{T_n(\xi) U_k(\xi_0) \sqrt{1-\xi_0^2}}{\sqrt{1-\xi^2}} K(\xi-\xi_0)\, d\xi d\xi_0$$

$$C_{nk}^* = \frac{2}{\pi^2} \int_{-1}^{1}\int_{-1}^{1} \frac{T_n(\xi) U_k(\xi_0) \sqrt{1-\xi_0^2}}{\sqrt{1-\xi^2}} K_*(\xi-\xi_0)\, d\xi d\xi_0$$

$$H_k = \frac{2}{\pi} \int_{-1}^{1} H(\xi_0) U_k(\xi_0) \sqrt{1-\xi_0^2}\, d\xi_0$$

The coefficients C_{nk}, C_{nk}^* can be represented explicitly. By using (2.8) and (2.4) we have

$$C_{nk} = \sum_{j=0}^{\infty} \left(\frac{\lambda}{2}\right)^{2j+2} K_j \alpha_{jnk}, \quad C_{nk}^* = \sum_{j=0}^{\infty} \left(\frac{\lambda}{2}\right)^{2j+2} K_j^* \alpha_{jnk} \tag{2.10}$$

$$\alpha_{jnk} = 2 \sum_{s=0}^{2j+1} (-1)^s \frac{(2j+1)!}{s!\,(2j+1-s)!}\, a_{k,s} b_{n,2j-s+1}$$

As can be noted, only the quantities $\alpha_{j,2n,2k+1}$ and $\alpha_{j,2n+1,2k}$ differ from zero. The system (2.9) determines the solution of the problem completely. In conclusion, let us note that the substitution $\xi = \cos\vartheta$ gives the representation (2.7) the customary form of a Fourier series in $\cos k\vartheta$ (without the zero term).

3. Stress intensity factors [8, 9]. Let us consider a plate with a doubly-periodic system of cracks. We assume

$$F(x) = 0, \quad F_1(x) = 0, \quad x \in [-1, 1] \tag{3.1}$$

Taking account of the third formula in (1.2) and the behavior of the Cauchy type integrals in (1.4) at the ends of the lines of integration [10], we find the stress intensity factors at the left and right ends of the cracks:

$$\partial_1 + i\partial_2 = \lim_{x\to l+0} \{(\sigma_y + i\tau_{xy}) \sqrt{2\pi(x-l)}\} = \tag{3.2}$$

$$i\sqrt{\pi l\, \theta_0(1)} = i\sqrt{\pi l} \sum_{k=1}^{\infty} \bar{A}_k$$

$$\partial_1^* + i\partial_2^* = \lim_{x\to -l-0} \{(\sigma_y + i\tau_{xy}) \sqrt{-2\pi(x+l)}\} =$$

$$i\sqrt{\pi l\, \theta_0(-1)} = i\sqrt{\pi l} \sum_{k=1}^{\infty} (-1)^k \bar{A}_k$$

Results of computations are given in Fig. 2. Curves of the change in the stress intensity factors ∂_i versus the relative size of the dommain $\lambda = 2l/\omega_1$ ($\omega_2 = \omega_1 e^{i\pi/3}$) are presented here. It was assumed that $\mu = 0.3$ in the computations.

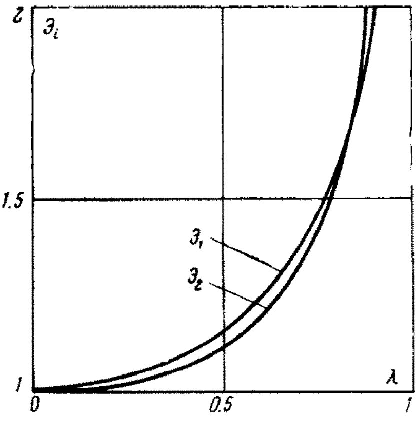

Fig. 2

4. Macroscopic lattice parameters. Let us find the law connecting the mean strains and the mean stresses in the lattice. This law is the Hooke's law for a continuous anisotropic medium having the same tension stiffness as the latice [5].

We obtain the increment in the displacements in the lattice upon going from the point z to its congruent point $z + \omega_\nu$ ($\nu = 1, 2$) by taking account of the last formula in (1.2) and the relationships (1.7). We have

$$2G[u(z+\omega_1) - u(z)] = {}^1/_4 (\sigma_1 + \sigma_2)(\varkappa + 1)\omega_1 - \omega_1\sigma_2 \tag{4.1}$$

$$2G[v(z+\omega_1) - v(z)] = \omega_1\tau + (\varkappa+1)\,\mathrm{Im}\,(a\delta_1)$$

$$2G[u(z+\omega_2) - u(z)] = \frac{1}{4}(\sigma_1+\sigma_2)(\varkappa+1)h + H\tau - h\sigma_2 -$$

$$\frac{\varkappa+1}{\omega_1}[H\,\mathrm{Im}\,(a\delta_1) - 2\pi\,\mathrm{Im}\,a], \qquad h = \mathrm{Re}\,\omega_2$$

$$2G[v(z+\omega_2) - v(z)] = \frac{1}{4}(\sigma_1+\sigma_2)(\varkappa+1)H + h\tau - H\sigma_1 +$$

$$\frac{\varkappa+1}{\omega_1}[h\,\mathrm{Im}\,(a\delta_1) - 2\pi\,\mathrm{Re}\,a], \qquad H = \mathrm{Im}\,\omega_2$$

On the other hand, the mean strains e_1, e_2, e_{12} and the angle of rotation of the cell ω in the lattice are related to the displacement increments as follows:

$$e_1\omega_1 = u(z+\omega_1) - u(z), \quad (e_{12}+\omega)\omega_1 = v(z+\omega_1) - v(z) \tag{4.2}$$

$$e_1 h + (e_{12} - \omega)H = u(z+\omega_2) - u(z)$$

$$e_2 H + (e_{12} + \omega)h = v(z+\omega_2) - v(z)$$

We assume below that the edges of the cracks are free of tractions.

Let $\theta_1(\xi)$ be the solution of (2.4) for $H(\xi) = -1$, and $\theta_2(\xi)$ be the solution of this equation for $H(\xi) = i$. Then the general solution $\theta(\xi)$ and its corresponding functional a are written as

$$\theta(\xi) = \sigma_2\theta_1(\xi) + \tau\theta_2(\xi), \quad a = l^2(a_1\sigma_2 + a_2\tau) \tag{4.3}$$

$$a_j = \frac{1}{2\pi i}\int_{-1}^{1} \xi\theta_j(\xi)\,d\xi, \qquad j = 1, 2$$

Substituting the displacement increments from (4.2) into (4.1) and taking account of (4.3), we arrive at the law connecting the mean stresses and the mean strains in the lattice, and also at a formula for the angle of rotation of the fundamental cell

$$e_1 = \frac{1}{E}(\sigma_1 - \mu\sigma_2) \tag{4.4}$$

$$e_2 = -\frac{\mu}{E}\sigma_1 + \frac{1}{E}\left(1 - \frac{2\pi\lambda^2}{H}\,\omega_1 \operatorname{Re} a_1\right)\sigma_2 - \frac{2\pi\lambda^2}{EH}\omega_1 \operatorname{Re} a_2\tau$$

$$2e_{12} = \frac{2\pi\lambda^2}{EH}\,\omega_1 \operatorname{Im} a_1\sigma_2 + \left(2\,\frac{1+\mu}{E} + \frac{2\pi\lambda^2}{EH}\,\omega_1 \operatorname{Im} a_2\right)\tau$$

$$\omega = \frac{\lambda^2\omega_1}{EH}\operatorname{Im}\left[(\delta_1 H - \pi)(a_1\sigma_2 + a_2\tau)\right] \tag{4.5}$$

The coefficients of σ_1, σ_2 and τ in (4.4) are macroscopic elastic lattice parameters. It can be shown that the matrix of the macroscopic parameters is symmetric ($\operatorname{Im} a_1 = -\operatorname{Re} a_2$) and energetically admissible [11].

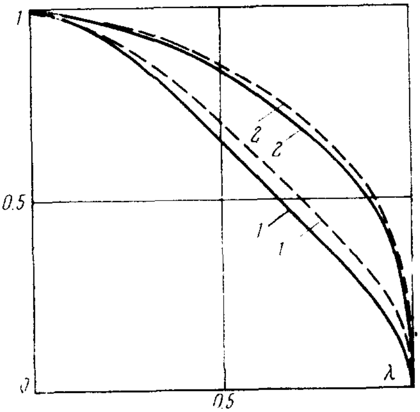

Fig. 3

Because of (2.6), (2.7) and (4.3), the functionals a_1 and a_2 in (4.4) can be expressed in terms of the first coefficient in the representation (2.7) by means of the formula

$$a_1 = \frac{1}{4i}A_1^{(1)}, \qquad a_2 = \frac{1}{4i}A_1^{(2)} \tag{4.6}$$

The superscript 1 here corresponds to the solution of the system (2.9) for $H_0 = -1$, $H_k = 0$ $(k = 1, 2, \ldots)$, and the superscript 2 corresponds to the solution of this system for $H_0 = i$, $H_k = 0$.

The following theorem summarizes all the above. An unbounded plane isotropic medium weakened by a doubly-periodic system of rectilinear cracks is identical "in the mean" to a special anisotropic medium controlled by the law (4.4).

The lattice under consideration can be interpreted as the model of the medium (4.4) and conversely.

The graphs of the macroscopic parameters

$$\frac{\langle E_2\rangle}{E} = \frac{1}{E\langle a_{22}\rangle} = \left\{1 - \frac{2\pi\lambda^2}{H}\,\omega_1 \operatorname{Re} a_1\right\}^{-1}$$

$$\frac{\langle G\rangle}{G} = \frac{1}{G\langle a_{66}\rangle} = \left\{1 + \frac{\pi\lambda^2}{H(1+\mu)}\,\omega_1 \operatorname{Im} a_2\right\}^{-1}$$

for a regular triangular (solid curves) and a square (dashes) lattices correspond to curves 1 and 2 in Fig. 3 (curves 2 practically coincide for both lattices). In this case the model medium is evidently orthotropic, i.e. $\langle a_{26} \rangle = \langle a_{62} \rangle = 0$

In conclusion, let us note that for $\lambda = 0$, $|\omega_1| = |\omega_2|$ we obtain a medium with one crack; as $|\omega_2| \to \infty$ we have a periodic system of cracks along the x-axis, and finally, for $|\omega_1| \to \infty$ and finite ω_2 we obtain a medium with a periodic system of parallel cracks.

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